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Published in:
Proceeding of the second international conference on education, technology, and sciences: Integrating technology and sciences into early and primary education

Publication date:
2017

Document version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Putra, Z. H., & Witri, G. (2017). Anthropological theory of the didactic: A new research perspective on didactic mathematics in Indonesia. In S. Syahrial, S. Saharudin, & J. Marzal (Eds.), *Proceeding of the second international conference on education, technology, and sciences: Integrating technology and sciences into early and primary education* (pp. 142-149). [15] Fakultas Keguruan dan Ilmu Pendidikan, Universitas Jambi, Indonesia.

ANTHROPOLOGICAL THEORY OF THE DIDACTIC: A NEW RESEARCH PERSPECTIVE ON DIDACTIC MATHEMATICS IN INDONESIA

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ABSTRACT

Théorie Anthropologique du Didactique/Anthropological Theory of the Didactic (ATD) is a new theory on didactic mathematics that was introduced by a French mathematician, Chevellard, in 1991. The ATD is an epistemological model of mathematical knowledge that can be applied to investigate human mathematical activities. Chevallard (1992) identified two aspects of a human mathematical activity that are a practical block and a knowledge block. Both are the main component of praxeologies. The practical block consists of a type of task (T) and a technique (τ). The type of task (T) is a task given to pupils, and they need a technique (τ) to solve it. Meanwhile, the theoretical block consists of a technology (θ) to explain the practical block, and a theory (Θ) is used to justify the technology (θ). The four elements (T, τ, θ, Θ) are connected. In this paper, we try to describe two cases based on ATD especially praxeologies. The first case is a research by Putra (2016) about elementary teachers' knowledge in designing contextual problems related to the multiplication of fractions. This study focused on the analysis of didactical praxeologies because it gave more attention to the mathematical didactics of teachers' representations from abstract to contextual problems. The second case is a study done by Wijayanti (2015). She analysed how ratio and proportion present on Indonesian mathematical textbooks. She tried to describe mathematical praxeologies of ratio and proportion of arithmetic and geometry. The implication of both studies is that the ATD especially praxeologies can be a model to analyse both mathematical and didactical knowledge. Therefore, we suggest researchers apply this model as an alternative method to study teachers' knowledge and analysis mathematical textbooks.

Keywords: Anthropological Theory of the Didactic, Praxeologies, Practical Blocks, Theoretical Blocks.

INTRODUCTION

In the 1980s, Yves Chevallard, a mathematician, gave his first course on the *didactic transposition processes* in the first summer school in *didactic mathematics* in Chamrousse, France (Bosch & Gascón, 2006; Bosch & Gascón, 2014). He proposed a theory to explain that knowledge or mathematical objects transpose through a relation of humans in an institution (Chevallard, 1992). His theory is mostly known by the French-speaking community, and nowadays it is disseminated to other communities and known as *Anthropological Theory of the Didactic* (ATD).

The ATD is a theory to observe human mathematical activities through an *epistemological model* of mathematical knowledge (Chevallard, 1992). Some frameworks and methods have been developed and applied through various studies in didactic mathematics. One of them is the notion of *praxeologies* (Durand-Guerrier, Winsløw & Yoshida, 2010; Hardy, 2009; Putra, 2016; Wijayanti, 2015) that can be used as a model to study mathematical and didactical knowledge.

In this paper, we describe two cases based on the *praxeologies*. The first case is a research by Putra (2016) about elementary teachers' knowledge in designing contextual problems related to the multiplication of fractions. This study focused on the analysis of *didactical praxeologies*, mathematical didactics of teachers' representations from abstract to contextual problems. The second case is a study done by Wijayanti (2015). This study analysed how ratio and proportion present on Indonesian lower secondary school mathematical textbooks. She described *mathematical praxeologies* of ratio and proportion of arithmetic and geometry.

Anthropological Theory of the Didactic (ATD)

The ATD proposes a model to describe mathematical knowledge of human activities through *praxeologies*. A *praxeology* consists of two components, *praxis* and *logos* (figure 1). The *praxis* or *practical block* consists of two parts, a *type of task* (T) and *technique* (τ). The *type of task* (T) is a specific kind of problems given to the learners. In the setting of classroom teaching and learning process at the elementary school level, the task can be taken from a mathematical textbook such as adding fractions. The pupils need a *technique* (τ) to solve the task, for instance, changing each fraction into the same denominator and adding numerators. Then, the *logos* or *knowledge block* comes from a Greek word (Chevallard, 2006) that refer to human thinking and reasoning about the cosmos. It also consists of two parts, a *technology* (θ) and a *theory* (Θ). The *technology* (θ) is about the justification for the *technique* (τ) by the pupils to solve the task. They justify that two fractions with different units can be added when those have the same unit. Meanwhile, arithmetic of fractions plays as a *theory* (Θ) to explain the the *technology* (θ). Those four elements (T, τ, θ, Θ) are used as a holistic model to study human knowledge.

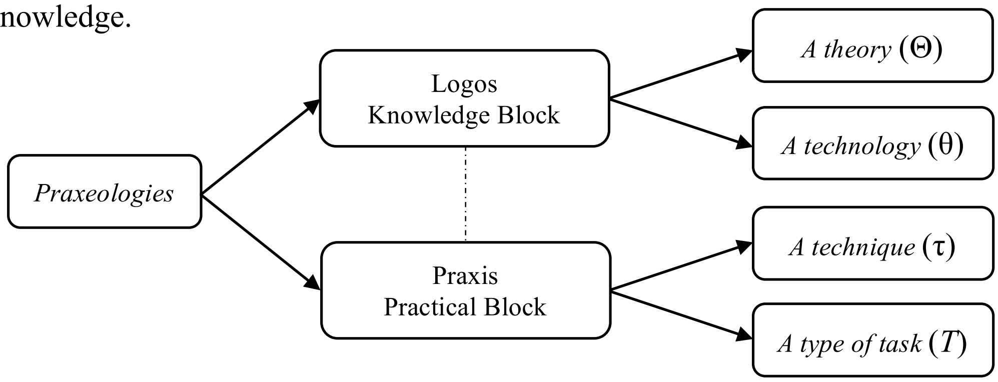

Figure 1. *A praxeological model*

Mostly a *type of task* (T) can be solved by various techniques, and a *technology* (θ) can employe some kinds of techniques. An organisation of a type of task (T) and techniques to solve that task is called as a *punctual organisation*. A common technology justifies several techniques for some types of tasks, then it becomes a *local organisation*. Since a theory (Θ) is often used for several technologies, it is called as a *regional organisation*. In fact, a *mathematical organisation* is a collection of *praxeologies* that belongs to a domain such as arithmetics.

The *praxeologies* do not only use to model and analyse mathematical knowledge but also didactical knowledge. The type of task (T) of didactical *praxeologies* is about how teachers teach mathematics such as how they organise a mathematical classroom situation for pupils to apply some techniques to solve a task, for instance, addition of fractions. The didactical techniques are also varied among teachers. Some of them probably propose a direct instruction from a mathematical technique they know or provide a contextual problem related to the task. In fact, *technology-theoretical* blocks of didactical *praxeologies* to justify the techniques are also varied based on their experiences and knowledge. An organisation of didactical *praxeologies* is known as a didactical organisation.

Case 1: Elementary teachers' knowledge

In this study, Putra (2016) gave a type of didactical task, constructing a contextual problem on fraction multiplications, to 50 Indonesian in-service elementary teachers who were taking a bachelor degree at Elementary School Teacher Education study program, University of Riau, in 2015. They were asked to pose a contextual problem for multiplication of a fraction by a whole number $\left(\frac{1}{2}\times 2\right)$ and multiplication of a fraction by a fraction $\left(\frac{1}{2}\times\frac{3}{4}\right)$. The type of tasks for both can be written generally as follows:

T_1 : given $\frac{a}{b}\times c$, design a contextual problem related to this equation.

T_2 : given $\frac{a}{b}\times\frac{c}{d}$, design a contextual problem related to this equation.

The teachers gave 2 types of correct answers and 4 types of incorrect answers. The first type of correct answers was designing a contextual problem based on a part-whole relationship. The example of the correct answer based on the part-whole relationship is "*A father has 2 hectares of land. ½ of this land is given to his cousin. How much land does the father now have?*". The second type of correct answer was designing a contextual problem based on measurement of area, for instance, a teacher wrote "*Andi would like to draw his land into a rectangle with ½ m long and 2 m wide. What is the area of the rectangle?*". Meanwhile, the 4 types of incorrect answers were constructing contextual problems based on repeated addition, an addition of fraction, a division of integer, and multiplicative comparison. A teacher gave an example based on the repeated addition as "*Dina has 2 packs of rice. Each pack contains ½ kg of rice. How much rice does Dina have?*" and based on the division of integers as "*A sister has 2 apples. Those apples will be given to two of her young brothers. How many apples will be got by each brother?*"

ISBN: 978-602-71682-1-3

The analysis for the first answer is that the teacher considered 2 as a whole and ½ is a part of whole, so s/he probably applied this *technique* to construct a contextual problem for multiplication of a fraction by a whole number. S/he interpreted the sign of "×" as "a part of". Actually, the answer does not only present $\frac{1}{2}\times 2$, but it can be interpreted as $2-\left(\frac{1}{2}\times 2\right)$. The second correct answer to construct a contextual problem based on the *technique* that ½ and 2 represent length and width of a rectangle, and use the formula of length × width to find the area of a rectangle. Even the answer is correct, but it is not really an appropriate unit (meter) to draw a rectangle in a paper.

When we analyse the two examples of incorrect answers, the first one is the *technique* based on the repeated addition that can be formulated as $\frac{1}{2}+\frac{1}{2}=2\times\frac{1}{2}$. Even though the answer for this contextual problem gives the same result with the multiplication of a fraction by a whole number, it has different technological reasoning. Meanwhile, the last answer is totally about the *technique* of division of integers as 2 is divided by 2.

There are three different types of answers for the task of type T_2. The first two types are categorised as correct answers based on measurement of area and part of a fraction. The teacher gave examples respectively as "*A rectangle is ½ m long and ¾ m wide. What is the area of the rectangle?*" and "*An aunt has ¾ part of a cake. ½ of that cake will be given to Ani. How much cake will Ani get?*". The incorrect answer is based on subtraction of fractions. A teacher wrote "*A mother wants to make a cake with ½ kg of flour and ¾ kg of sugar. How much other materials are needed if the total weight of the cake should be 3 kg?*".

The analysis for the *techniques* to explain the correct answers is almost similar to the previous type of tasks. The teacher still chose an appropriate unit (meter) because it will be a problem for pupils when they try to draw a rectangle. It will be better if they use a unit such as centimeter or decimeter, so they can perfectly draw the rectangle in a paper and find the area. The second correct example is a contextual problem based on the *technique* of part of a fraction or sometimes known as a part-part relationship. Meanwhile, the incorrect answer can be formulated as 3 - (½ + ¾). This *technique* is totally away from the task of multiplication of fractions.

Case 2: Lower secondary school mathematics textbooks

The second case is about an analysis of ratio and proportion presented in lower secondary school mathematics textbooks. This study was conducted by Wijayanti (2015) in order to show the link between proportion in geometry (similarity) and arithmetic (ration and proportion). She analysed examples and exercises from 6 common Indonesian textbooks for grade 7 and grade 9 through mathematical *praxeologies* specifically *types of task* (T) and possible *techniques* (τ) to solve the tasks.

Wijayanti (2015) defined 3 different types of tasks for arithmetic in common textbooks. The first one is T_1^{Ar} : given $(x_1, ..., x_n)$ and $(y_1, ..., y_n)$ decide if $(x_1, ..., x_n) \sim (y_1, ..., y_n)$. The second type of task is T_2^{Ar} : given $(x_1, ..., x_n)$ and $(y_1, ..., y_n)$ compare $\frac{x_i}{x_j}$ for $i = 1, ..., n$, and the third one is T_3^{Ar} : given $(x_1, ..., x_n)$, y_1 find $y_2, ..., y_n$ so that $(x_1, ..., x_n) \sim (y_1, ..., y_n)$. Meanwhile, she defined 2 common types of tasks for geometry in those textbooks. The first type of task is closed related to T_1^{Ar} and it is stated as T_1^{Gr} : Given two polygons with the same angles and also given the side lengths of two polygons that correspond to, decide if the polygons are similar, and the second type of task is closed related to T_3^{Ar}, and it is defined as T_3^{Gr} : given similar figures with corresponding sides (x_1, ..., x_n) and (y_1, ..., y_n) with x_1, ..., x_n and y_1 known, find the unknown sides y_2, ..., y_n.

Among 6 common Indonesian textbooks, the proportion in arithmetics is introduced in grade 7, and in geometry in grade 9 (Wijayanti, 2015). The actual tasks in the textbooks for arithmetics are usually given trough contextual problems such as scales, speeds, and measurement of area and others. Meanwhile, the tasks for geometry commonly state as comparing the length of geometry figures. In Wijayanti's study, she found that the common types of tasks on arithmetics appeared on the textbooks based and T_3^{Ar} and T_1^{Gr} for geometry.

We would like to give two examples from Indonesian textbooks. The first example we take from a mathematical textbook for grade 7 written by Wintarti et al., (2008, pp.142). The task is written as "*two pupils can carry 15 books. How many books can 8 pupils carry?*" In this task, the writers proposed two mathematical techniques as follows:

Table1: *Technique 1*

Number of pupils	Numbers of Books
2	15
4	30
8	60

Tabel 2: *Technique 2*

Number of pupils	Numbers of Books
2	15
2	15
2	15
2	15
8	60

Wijayanti (2015) categorised this tasks as T_3^{Ar} because it can be written as t^{Ar} : given (2, 8), 15 find y, so that (2, 8) ~ (15, y). Meanwhile, the technique 1 proposed by the writers is based on multiplicative reasoning that we can interpret as τ_1 : multiply 2 by 2 and 15 by 2, and we get 4 and 30, and then multiply 4 by 2 and 30 by 2, and we get 8 and 60. Meanwhile, the technique 2 is based on repeated addition that it can be interpreted as τ_2 : 2 ~ 15, 2+2+2+2 2 ~ 15+15+15+15, so 8 ~ 60.

The second example is taken from a mathematical textbook for grade 9 written by Wagiyo, Mulyona & Susanto (2008). The task is written as *"given two similar triangles that can be seen in the figure below (*Figure 2). *Determine the length of x and y?*"

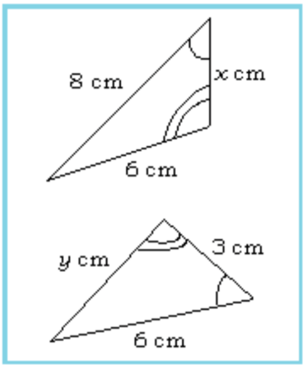

Figure 2. *Two similar triangles*

The writers provided a technique as follows:

Since both triangles are similar, corresponding sides have the same ratio that can be written as:

$$\frac{8}{6} = \frac{6}{y} = \frac{x}{3} \text{ or } \frac{6}{8} = \frac{y}{6} = \frac{3}{x}$$

$$\frac{8}{6} = \frac{6}{y} \longleftrightarrow 8y = 36$$

$$y = \frac{36}{8}$$

$$y = 4\frac{1}{2}$$

$$\frac{8}{6} = \frac{x}{3} \longleftrightarrow 6y = 24$$

$$y = \frac{24}{6}$$

$$y = 4$$

This task can be categorised as T_3^{Gr} and can be written as t^{Gr} = given similar figures with corresponding sides (8, 6, x) and (6, y, 3). Find the unknown sides x and y. the technique was proposed by the writers is categorised as algebraic manipulation that it can be written as τ_3 : if $\frac{x_1}{x_2} = \frac{y_1}{y_2}$, $x_1 y_2 = x_2 y_1$, so $y_2 = \frac{x_2 y_1}{x_1}$.

DISCUSSION

We give two different cases how the ATD through *praxeologies* plays as a framework to study mathematical and didactical situations in Indonesian contexts. The first case study focused on in-service teachers' didactical knowledge through constructing meaningful mathematical problems for multiplication of fractions. There are two common correct mathematical techniques for the type of task T_1 and T_2. The techniques are based on the part-whole/part-part relationship and the measurement of area.

The teachers proposed more incorrect techniques for the task of type T_1 than T_2. However, there were no answers proposed by teachers based on ratio and proportion. For instance, *a metal bar 2 kg weight has 1-meter long. What is the weight of a similar bar that is* $\frac{1}{2}$*-meter long.* One important result from the study of elementary teachers' knowledge on constructing contextual problems for multiplication of fraction is that the teachers proposed answers based on their mathematical knowledge. It means that the didactical knowledge is thus closely related to a mathematical knowledge because the didactical knowledge is about a knowledge of teaching mathematical.

From the study of lower secondary textbook analysis on ratio and proportion, it seems that there is no type of task on geometry corresponding to T_2^{Ar} appeared in those textbooks. Actually, a task to enlarge a geometrical figure such as a rectangle can be interpreted as T_2^{Gr} because a pupil needs to know the ratio between width and length of that rectangle. Meanwhile, the techniques proposed by writers for the two examples are totally different between arithmetic and geometry. The techniques for the arithmetic task is less formal that geometrical task. The algebraic technique applied in the geometrical task can be used to solve the arithmetic one, and vice versa. Since we just analysed two examples from two lower secondary textbooks, we assumed that there must be some examples proposed more than one or two techniques, and the techniques will be varied when pupils try to solve those tasks.

CONCLUSION

The ATD through *praxeologies* provides a model to study mathematical and didactical knowledge. The *praxeologis* especially practical block can be used directly to model tasks given to pupils or teachers, or tasks appeared on textbooks. From a type of tasks, we can model some possible techniques that can support pupils learning process. Actually, those two studies just an example of research on didactic mathematics in the context of Indonesia. We hope that these can inspire other researchers to do researches based on the ATD in Indonesia.

REFERENCES

Bosch, M., & Gascón, J. (2006). Twenty-five years of the didactic transposition. *ICMI Bulletin*, *58*, 51-65.

Bosch, M., & Gascón, J. (2014). Introduction to the Anthropological Theory of the Didactic (ATD). In A. Bikner-Ahsbahs & S. Prediger (Eds.) *Networking of theories as a research practice in mathematics education* (pp. 67-83). Springer International Publishing.

Chevallard Y. (1992). Fundamental concepts in didactics: Perspectives provided by an anthropological approach. In: Douady R, Mercier A (eds) Research in Didactique of Mathematics. Selected Papers. La Pensée sauvage, Grenoble, France, pp 131-167.

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In *Proceedings of the 4th Conference of the European Society for Research in Mathematics Education (CERME 4)* (pp. 21-30). Barcelona, Spain: FUNDEMI-IQS.

Durand-Guerrier, V., Winsløw, C. & Yoshida, H. (2010). A model of mathematics teacher knowledge and a comparative study in Denmark, France and Japan. *Annales de didactique et des sciences cognitives*, 15, 147-172.

Hardy, N. (2009). Students' perceptions of institutional practices: the case of limits of functions in college level calculus courses. *Educational Studies in Mathematics*, *72*(3), 341-358.

Putra, Z. H. (2016). Evaluation of elementary teachers' knowledge on fraction multiplication using anthropological theory of the didactic. Presented in 13th International Congress on Mathematical Education, Hamburg, Germany.

Wagiyo, A., Mulyono, S., & Susanto. (2008). *Pegangan belajar matematika 3: untuk SMP/MTs kelas IX*. Pusat Perbukuan, Department Pendidikan Nasional, Jakarta.

Wijayanti, D. (2015). Relating arithmetical techniques of proportion to geometry: the case of Indonesian texbooks. In: Nada Vondrova (Eds), *Proceeding of the ninth congress of the European Society for Research in Mathematics Education* (pp.3157-3163). Prague, Czech Republic.

Wintarti, A., Rahaju, E. B., Sulaiman, R., Yakob, C, & Kusrini. (2008). Contextual teaching and learning matematika: Sekolah menegah pertama/ madrasah tsanawiyah kelas VII edisi 4. Pusat Perbukuan, Department Pendidikan Nasional, Jakarta.